

1) Model

- Price S_t and variance v_t evolve together.
 - v_t mean-reverts to θ at speed κ .
 - Shocks to S_t and v_t are correlated by ρ (leverage).
- A European call's fair value is the risk-neutral expectation

$$C(S_0, K, T) = e^{-rT} \mathbb{E}[(S_T - K)^+].$$

- The code computes this expectation by **Monte Carlo** under Heston (many simulated paths), using a stable **full-truncation** step so $v_t \geq 0$.

2) Anchoring & calibration (fast and light)

- Estimate **ATM IV** from the chain; set $v_0 \approx \theta \approx \text{ATM}^2$.
- Fit the remaining shape with a small grid over σ_v (vol-of-vol) and ρ : choose (σ_v, ρ) that **minimizes RMSE** between model prices and market mids on a handful of ≈ 30 -day strikes around ATM.

3) What gets traded (mispricing)

- For each ≈ 30 -day **call** in the chain:
 - compute **model price** C_{mod} by MC, get **market mid** C_{mkt} .
 - **Undervalued** if $C_{\text{mod}} \geq (1 + \tau)C_{\text{mkt}} \rightarrow$ **buy** the call.
 - **Overvalued** if $C_{\text{mod}} \leq (1 - \tau)C_{\text{mkt}} \rightarrow$ **sell (write)** the call.
- A small cap on buys/writes controls exposure per bar.

4) Practical glue

- Pick the slice by days-to-expiry ≈ 30 and start near **ATM** via a target **delta** ($\Delta \approx 0.5$).
- Use the broker's option chain (contract functions) to select the nearest **listed** strike and to send the order.

Essence

- **Theory**: price = discounted expectation under Heston.
- **Inference**: infer $(v_0, \theta, \sigma_v, \rho)$ from the smile (light grid, ATM anchor).
- **Action**: go long cheap, short rich, within one maturity bucket.
- **Why Heston**: it naturally captures skew, clustering, and the price/volatility link (ρ).